Chapter 55
Analysis of Multipath Parameter Estimation Accuracy in MEDLL Algorithm

Yuan Gao, Feng Liu and Teng Long

Abstract MEDLL (Multipath Estimating Delay Lock Loop) is an excellent multipath mitigation algorithm, the core of the algorithm is estimated multipath parameters, and parameter estimation accuracy determines the actual performance of the algorithm. Previous studies and experiments have demonstrated the anti-multipath MEDLL algorithm performance, lack of research but for its observations on the multipath performance, multipath parameter estimation performance. Studied MEDLL algorithm the multipath estimation accuracy and thermal noise, front-end bandwidth, the correlator spacing and sampling rate and other parameters of the relationship obtained MEDLL algorithm under different conditions, the accuracy of the estimate of the multipath parameters.

Keywords MEDLL · Multipath parameter estimation · GNSS receiver

55.1 Introduction

Improvements due to GNSS (Global Navigation Satellite System) augmentations and GNSS modernization are reducing many source of error leaving and shadowing as significant and sometimes dominant contributors to error [1]. As a result, the multipath rejection performance of GNSS receiver becomes more and more important. The MEDLL (Multipath Estimating Delay Lock Loop) algorithm is a multipath mitigation algorithm which was first proposed in Ref. [2].

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The performance of MEDLL algorithm applied in NovAtel GPS receiver is analysed in Ref. [3, 4]. It has been proved the MEDLL algorithm is an effective way to mitigate errors caused by multipath. The basic idea of MEDLL algorithm is mitigating multipath by separating direct signal from multipath by estimating parameters of multipath. As a result, MEDLL algorithm is also applied in multipath monitoring [5].

From above we know previous studies are mainly concentrated in performance of MEDLL algorithm and how to improve the performance. Some studies are also focused on how to reduce resource consumption due to highly complexity of MEDLL algorithm. However, this is rare study on multipath parameter estimation accuracy. In fact, multipath parameter estimation whose accuracy determines performance of multipath mitigation is the key to MEDLL algorithm especially in application using MEDLL algorithm for multipath monitoring.

55.2 Multipath Models and MEDLL Algorithm

55.2.1 Multipath Signal Models

Multipath is the reception of reflected or diffracted of the desired signal as shown in Fig 55.1. Multipath is divided into two kinds: reflected one and scattering on. Scattering signal often performs as an additional noise channel, which has little effects on GNSS receiver working. Therefore, only reflected signal is in consideration when studying multipath parameter estimation accuracy. The reflected signal can be approximately considered as the direct signal with changes of amplitude, phase and time delay. Then a simple model for the complex envelope of a received signal $r(t)$ with multipath after frequency down conversion is

$$r(t) = \sum_{n=0}^{N} A_n e^{j(2\pi fct+\phi_n)} p(t-\tau_n) + n_r(t) \tag{55.1}$$

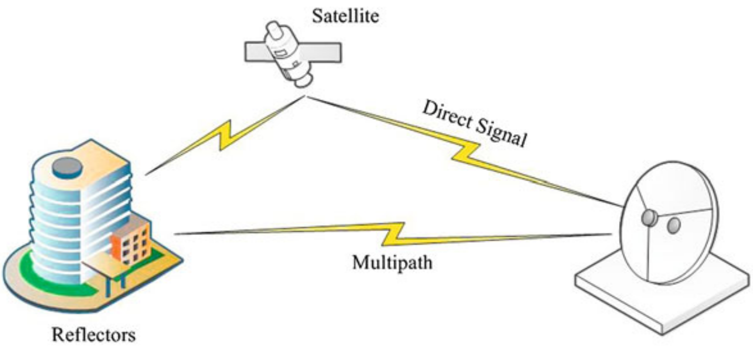

Fig. 55.1 Generation of multipath

where, $p(t)$ is the spreading code of the transmission signal, f is the carrier frequency, A_n is the amplitude of the nth signal, ϕ_n is the carrier phase of the nth signal, τ_n is the time delay of nth signal, when $n = 0$, it means signal is the direct one; $n_r(t)$ is the complex envelope of receiver front-end noise; it is a narrowband random process which can be represented by the quasi-sinusoidal form as shown in Eq. (55.2).

$$n_r(t) = [n_{rc}(t) + jn_{rs}(t)]e^{j(2\pi fct+\phi_0)} \tag{55.2}$$

It's known from Eq. (55.2) that one multipath signal will be determined and separated from the direct signal once parameters of amplitude, time delay and carrier phase has been calculated by estimating. Therefore, the main task of MEDLL algorithm is determining these three parameters of multipath.

55.2.2 Principle of MEDLL Algorithm

MEDLL loop which is a group of correlator with different local replica code phase is the hard foundation of performing MEDLL algorithm. MEDLL loop is shown in Fig. 55.2. A group of

The equation of $s(t)$ can be obtained from Eq. (55.1) assuming the carrier frequency has been estimated accurately during carrier stripping.

$$s(t) = \sum_{n=0}^{N} A_n e^{j\phi_n} p(t - \tau_n) + n(t), n(t) = [n_c(t) + jn_s(t)]e^{j\phi_o} \tag{55.3}$$

This group of integrated sum obtained from MEDLL loop which is called measured correlation function $R_s(\tau)$ is actually a group of measurement of $s(t)$. That is:

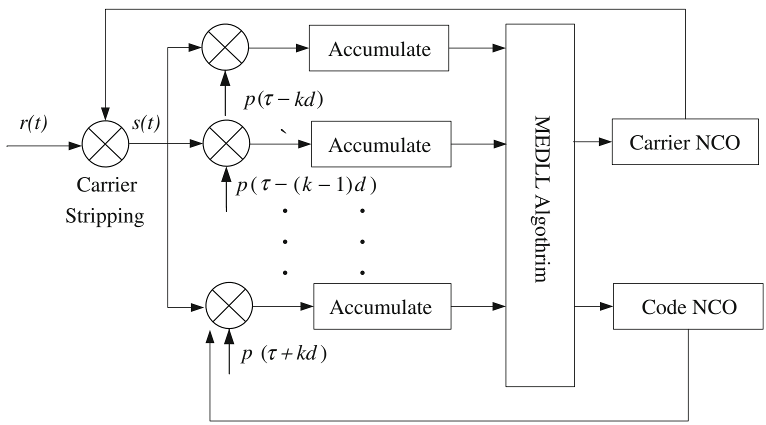

Fig. 55.2 MEDLL loop block diagram

$$R_s(\tau) = \sum_{n=1}^{k} [s(\tau) \otimes p(\tau + \tau_k)] \tag{55.4}$$

Assuming $\hat{N}$ set of parameters obtained from MEDLL algorithm is $(A_n, \tau_n, \phi_n)(n = 0, 1, \ldots \hat{N})$, $\hat{N}$ is estimated value of multipath signal's number. Ignoring noise, the estimated values $\hat{s}(t)$ of $s(t)$ is:

$$\hat{s}(t) = \sum_{n=0}^{\hat{N}} \hat{A}_n e^{j\hat{\phi}_n} x(t - \hat{\tau}_n), M(\hat{A}, \hat{\tau}, \hat{\phi}) = \int_{t-T_p}^{t} [s(t) - \hat{s}(t)]^2 dt \tag{55.5}$$

where, T_p is predetection correlation time and $M(\hat{A}, \hat{\tau}, \hat{\phi})$ is mean square error of $\hat{s}(t)$.

According to maximum likelihood estimation (MLE) rule, $M(\hat{A}, \hat{\tau}, \hat{\phi})$ should be minimum, that is:

$$\frac{\partial M\left(\hat{A}, \hat{\tau}, \hat{\phi}\right)}{\partial \hat{A}} = 0, \frac{\partial M\left(\hat{A}, \hat{\tau}, \hat{\phi}\right)}{\partial \hat{\tau}} = 0, \frac{\partial M\left(\hat{A}, \hat{\tau}, \hat{\phi}\right)}{\partial \hat{\phi}} = 0 \tag{55.6}$$

According references [2, 3], Eq. (55.6) can be solved:

$$\begin{array}{c} = \max \\ \hat{\tau}_i \end{array} \left\{ \mathrm{Re} \left[R_s(\tau) - \sum_{\substack{n=0 \\ n \neq i}}^{\hat{N}} \hat{A}_k R_{ref}(\tau - \hat{\tau}_k) e^{j\hat{\phi}_i} \right] \right\} \atop \hat{A}_i \tag{55.7}$$

where $R_{\mathrm{ref}}(\tau)$ which is named with reference correlation function in MEDLL algorithm is the correlation function of $p(t)$ in this receiver channel [2]. The reference correlation function reflects the characteristics of the receiver's channel.

55.2.3 Multipath Parameters Solving

It is hard to solve Eq. (55.7) directly because parameters are tightly coupled. Therefore, Eq. (55.7) is usually solved with method of iterative calculation.

We just take one multipath in consideration in order to simplify the analysis. That is:

$$s(t) = s_o(t) + s_1(t) \tag{55.8}$$

where, $s_0(t)$ is the direct signal with correlation function of $R_0(\tau)$ and $s_1(t)$ is the direct signal with correlation function of $R_1(\tau)$. Known by the linear characteristics of correlation function:

$$R_s(\tau) = R_0(\tau) + R_1(\tau) \tag{55.9}$$

The general iterative solver process is:

Step 1 Assuming $R_0(\tau) = R_s(\tau)$, the first set estimated parameters $(\hat{A}_0, \hat{\tau}_0, \hat{\phi}_0)_1$ of $R_0(\tau)$ will be obtained. Then, combining this set parameters with reference correlation function, the first estimated value $\hat{R}_0(\tau)_1$ of $R_0(\tau)$ will be obtained;

Step 2 Assuming $R_1(\tau) = R_s(\tau) - \hat{R}_0(\tau)_1$, the first set estimated parameters $\left(\hat{A}_1, \hat{\tau}_1, \hat{\phi}_1\right)_1$ of $R_1(\tau)$ will be obtained. Then, combining this set parameters with reference correlation function, the first estimated value $\hat{R}_1(\tau)_1$ of $R_0(\tau)$ will be obtained. Assuming $R_0(\tau) = R_s(\tau) - \hat{R}_1(\tau)_1$, the second set estimated parameters $\left(\hat{A}_0, \hat{\tau}_0, \hat{\phi}_0\right)_2$ of $R_0(\tau)$ will be obtained. Then, combining this set parameters with reference correlation function, the second estimated value $\hat{R}_0(\tau)_2$ of $R_0(\tau)$ will be obtained;

Step 3 Assuming $R_1(\tau) = R_s(\tau) - \hat{R}_0(\tau)_2$, the second set estimated parameters $\left(\hat{A}_1, \hat{\tau}_1, \hat{\phi}_1\right)_2$ of $R_1(\tau)$ will be obtained. Then, combining this set parameters with reference correlation function, the second estimated value $\hat{R}_1(\tau)_2$ of $R_1(\tau)$ will be obtained;

Repeat Step 2, 3 until the mean square $E(V_n)$ of iteration residuals V_n has met expectations. V_n is defined in Eq. (55.10)

$$V_n = R(\tau) - \hat{R}_0(\tau)_n - \hat{R}_1(\tau)_n, E(V_n) = \int_{-D}^{D} \left|R(\tau) - \hat{R}_0(\tau)_n - \hat{R}_1(\tau)_n\right|^2 d\tau \tag{55.10}$$

55.2.4 Algorithm of Parameters Estimation

Assuming that a correlation function $R_s(\tau)$ contains only one path signal, in order to estimate its parameters, we should first find the maximum energy point τ_{max} make Eq. (55.11) established.

$$\left\{[\mathrm{Re}(R_s(\tau_{max})]^2 + [\mathrm{Im}(R_s(\tau_{max})]^2\right\} = \max\left\{[\mathrm{Re}(R_s(\tau)]^2 + [\mathrm{Im}(R_s(\tau)]^2\right\} \tag{55.11}$$

Carrier phase estimated value is obtained through four quadrant arctangent at this point as shown in Eq. (55.12).

$$\hat{\phi}_s = arc\tan 2\left(\frac{\mathrm{Im}(R_s(\tau_{\max}))}{\mathrm{Re}(R_s(\tau_{\max}))}\right) \tag{55.12}$$

Then, at the same point time delay estimated value is obtained through incoherent early-late code discriminator as shown in Eq. (55.13).

$$\begin{aligned} I_E &= \mathrm{Re}(R_s(\tau_{\max} - dT_c)), Q_E = \mathrm{Im}(R_s(\tau_{\max} - dT_c)) \\ I_L &= \mathrm{Re}(R_s(\tau_{\max} + dT_c)), Q_L = \mathrm{Im}(R_s(\tau_{\max} + dT_c)) \\ \tau_{EMLP} &= \left[\left(I_E^2 + Q_E^2\right) - \left(I_L^2 + Q_L^2\right)\right] \end{aligned} \tag{55.13}$$

where, T_c is the chip length of spreading code.

At last, the amplitude estimated value is obtained through reference correlation function as shown in Eq. (55.14).

$$\hat{A}_s = \sqrt{\mathrm{Re}(R_s(\tau_{\max}))^2 + \mathrm{Im}(R_s(\tau_{\max}))^2}\,\frac{R_{ref}(0)}{R_{ref}(\hat{\tau}_s)} \tag{55.14}$$

55.3 Error Analysis of Multipath Parameters Estimation

It's known in Eq. (55.7) that time delay plays a more important part because it determines if the estimating point is right. On the other hand, time delay gets more concerned in applications which apply MEDLL algorithm into multipath mitigation (actually most applications are such). Therefore, in this paper estimation accuracy of time delay is mainly discussed.

55.3.1 Error Source of Estimation

It's known from Sect. 55.2.4 that estimation of time delay is similar to code tracking in normal GNSS receiver. Therefore, the source of estimation is mainly thermal noise, front-end bandwidth and correlator spacing [1, 6]. There is another source named with iteration residuals because iteration calculation is applied into.

Iteration residual is similar to multipath when estimating time delay. In the following analysis we first analyze other sources then take iteration residual into consideration.

55.3.2 Error Brought by Noise and Limited Bandwidth

Without consideration of iteration, assume the theoretical time delay in Eq. (55.13) is ε, and then the linear model of EMLP's output is:

$$\tau_{EMLP} = K\varepsilon_{\tau} + n_{\tau}, |\varepsilon_{\tau} - \tau_{EMLP}| \to 0 \tag{55.15}$$

where, K is slope of discriminator around theoretical value independent of noise. n_{τ} is the output noise of discriminator whose bandwidth is greater than predetection correlation bandwidth $1/T_{\mathrm{p}}$. As a result, n can be considered as zero mean and independent in each output, and then mean square of discrimination error is [6]:

$$\sigma_{EL}^2 = \frac{2N_{\tau}T_p}{K^2} \tag{55.16}$$

where, N_{τ} is power of n_{τ}. Then Eq. (55.13) is rewritten as follow:

$$\begin{aligned} I_E &= \mathrm{Re}\big(R_{ref}(\tau_{\max} - dT_c) + n_E\big), I_L = \mathrm{Re}\big(R_{ref}(\tau_{\max} + dT_c) + n_L\big) \\ Q_E &= \mathrm{Im}\big(R_{ref}(\tau_{\max} - dT_c) + n_E\big), Q_L = \mathrm{Im}\big(R_{ref}(\tau_{\max} + dT_c) + n_L\big) \end{aligned} \tag{55.17}$$

where, the noise term is obtained in following equation:

$$n_E = n(t) \otimes p(\tau_{\max} - dT_c), n_E = n(t) \otimes p(\tau_{\max} + dT_c) \tag{55.18}$$

Then the output of discriminator can be presented as follow:

$$\begin{aligned} \tau_{EMLP} &= \left[\left(I_E^2 + Q_E^2\right) - \left(I_L^2 + Q_L^2\right)\right] \\ &= \left|R_{ref}(\tau_{\max} - T_c d) + n_E\right|^2 - \left|R_{ref}(\tau_{\max} + T_c d) + n_L\right|^2 \\ &= \left|R_{ref}(\tau_{\max} - T_c d)\right|^2 - \left|R_{ref}(\tau_{\max} + T_c d)\right|^2 + |n_E|^2 - |n_L|^2 \\ &\quad + R_{ref}(\tau_{\max} - T_c d)n_E^* - R_{ref}(\tau_{\max} + T_c d)n_L^* \\ &\quad + R_{ref}^*(\tau_{\max} - Tcd)n_E - R_{ref}^*(\tau_{\max} + T_c d)n_L \end{aligned} \tag{55.19}$$

Because expectation of terms containing noise in above equation is zeros [7], the expectation of τ_{EMLP} is:

$$E(\tau_{EMLP}) = E\left(\left|R_{ref}(\tau_{\max} - T_c d)\right|^2 - \left|R_{ref}(\tau_{\max} + T_c d)\right|^2\right) \tag{55.20}$$

Then

$$K = E'(\tau_{EMLP})\big|_{\tau_{EMLP} = \tau_{\max}}, N_{\tau} = Var(\tau_{EMLP}) \tag{55.21}$$

Equations (55.19–55.21) substituted into Eq. (55.16) after finishing is [1, 6, 8]:

$$\sigma_{EL}^2 = \frac{\frac{1}{4\pi^2}\int_{-\beta_s/2}^{\beta_s/2} S_s(f)\sin^2(\pi f d T_c)df}{(P_s/N_0)\left(\int_{-\beta_s/2}^{\beta_s/2} f S_s(f)\sin(\pi f d T_c)df\right)^2} \times \left[1 + \frac{\int_{-\beta_s/2}^{\beta_s/2} S_s(f)\cos^2(\pi f d T_c)df}{(T_P P_s/N_0)\left(\int_{-\beta_s/2}^{\beta_s/2} S_s(f)\cos(\pi f d T_c)df\right)^2}\right] \tag{55.22}$$

where, $S_s(f)$ is transmit spectrum of navigation signal, P_s is received power, N_0 is single band power spectral (assuming noise is white), β_s is front-end bandwidth.

55.3.3 Error Brought by Iteration Residual

Because iteration residual is similar to multipath when estimating time delay, we assuming error brought by iteration residual is $\sigma_{ir}(V_n)$, that is:

$$\sigma_\tau = \sigma_{mp}(V_n) + \sigma_{EL} \tag{55.23}$$

According to MLE rules:

$$\frac{\partial}{\partial\tau_1}\int_{t-T_p}^{t}[s(t) - \hat{s}_0(t) - \hat{s}_1(t)]^2 dt = \frac{\partial}{\partial\tau_1}\int_{t-T_p}^{t}[V_n]^2 dt = 0 \tag{55.24}$$

From Eq. (55.24) we can know that V_N is considered independent of τ_1 when estimating delay τ_1 of $s_1(t)$, that is:

$$V_n = R_{ref}(\tau_0) - R_{ref}(\tau_0 + \sigma_{EL}), \sigma_\tau = \sigma_{mp}\big(R_{ref}(\tau_0) - R_{ref}(\tau_0 + \sigma_{EL})\big) + \sigma_{EL} \tag{55.25}$$

55.4 Analysis of Accuracy and Simulation

Correlation spacing d is usually tiny in MEDLL loop. Therefore, make $d \to 0$, known from Eq. (55.22), when received power is constant and $T_p \to \infty$,

$$\sigma_{EL}^2 \to \frac{1}{(2\pi)^2(P_\beta/N_0)\omega_{RMS}^2}$$
$$P_\beta = P_s\int_{-\beta_s/2}^{\beta_s/2} S_s(f)df,\ \omega_{RMS} = \sqrt{\int_{-\beta_s/2}^{\beta_s/2} f^2 S_s(f)df} \tag{55.26}$$

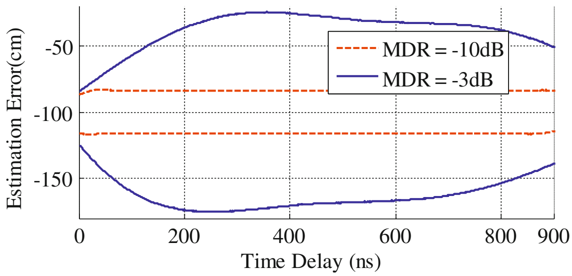

Fig. 55.3 Accuracy of GPS L1C/A code delay estimation under different MDR

where, ω_{RMS} is named with root-mean-squared bandwidth [1] or Gabor bandwidth [9]. It is useless when $d \to 0$ because of Gabor bandwidth.

Take BPSK-R(n) modulation which is used in most running satellite navigation system as an example. C/A code of GPS L1 is BPSK-R(1), when $d \to 0$:

$$\sigma_{EL}^2 \cong \frac{1}{2P_\beta/N_0\beta_s T_c}\left(1 + \frac{1}{T_p P_\beta/N_0}\right), \quad d \le \frac{1}{T_c\beta_s} \tag{55.27}$$

From above equation we know that when front-end bandwidth is determined and correlator spacing is smaller than $1/T_c\beta_s$ accuracy of time delay estimation tends to a value independent of correlator spacing. Figure 55.3 shows accuracy of GPS L1C/A code delay estimation under different MDR (Multipath-to-direct Ratio).

From Fig. 55.3 we can see that accuracy of time delay estimation is mainly determined by thermal noise and front-end bandwidth and tends to a constant value when MDR is small. However, when MDR is large that accuracy is mainly determined by iteration residual instead and its envelope tends to envelope of error caused by multipath.

55.5 Conclusions

In this paper, we first analyze the accuracy of multipath parameter estimation from principle and implementation of MEDLL. Then error source of estimation is analyzed and equation of accuracy is given and proved to be correct by simulation. Otherwise, the theoretical limit of accuracy is given under various conditions. It is helpful to design of application using MEDLL algorithm.

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